Space

Lemma (equality)

If $\langle v_1, w\rangle = \langle v_2, w\rangle$ for all w in an inner product space X then $v_1 = v_2$. then $\langle v_1, w\rangle = 0$ for all $w \in X \iff v_1 = 0$

Def (Sesquilinear form)

Let X and Y be vector spaces over the same field $K = (R$ or $C)$ then a sesquilinear form h on $X \times Y$ is a maping

$h : X \times Y \to K$

(a) $h(x_1 + x_2, y) = h(x_1, y) + h(x_2, y)$

$h(x, y_1 + y_2) = h(x, y_1) + h(x, y_2)$

R^3 is hilbert space

R^3 ka inner product change to dot product

$\alpha \in K = R$

(c) $h(\alpha x, y) = \alpha h(x, y)$ $\quad x_1, x_2 \in X$

$h(x, \beta y) = \bar{\beta} h(x, y)$ $\quad y_1, y_2 \in Y$

Theorem (Riesz representation)

$h : H_1 \times H_2 \to K$, $\quad \because H_1, H_2$ be Hilbert

a bounded sesquilinear form

$h(x, y) = \langle Sx, y \rangle$

where $S : H_1 \to H_2$ is a bounded linear operator. S is uniquely determined by h and

$\|S\| = \|h\|$

$\Rightarrow$ v.v.v.

dual space of the real space l^2 is l^2

Hilbert - adjoint operator T^*

def

let $T : H_1 \to H_2$ be a bounded linear operator where H_1 and H_2 are Hilbert spaces. Then the adjoint Hilbert operator T^* of T

$T^* : H_2 \to H_1$

$x \in H_1 \quad y \in H_2$

$(Tx, y) = (x, T^* y)$

Theorem (existence)

The Hilbert-adjoint operator T^* of T in exist is unique and is bounded linear operator with norm

$\|T^*\| = \|T\|$

e.g

Identity ka adjoint Hibret operator itself identit

Zero ka be itself

v.v. imp **Lema (zero operator)**

let X and Y be inner product space and $Q: X \to Y$ a bounded linear operator then

(a) $Q = 0$ iff $(Qx, y) = 0$ $x \in X$ $y \in Y$

(b) If $Q: X \to Y$ where X is complex and $(Qx, x) = 0$ $\forall\ x \in X$ then $Q = 0$

property of Hilbret adjoint property

let H_1, H_2 be Hilbret space $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar

(a) $(T^* y, x) = (y, Tx)$

(b) $(S+T)^* = S^* + T^*$

(c) $(\alpha T)^* = \bar{\alpha} T^*$

(d) $(T^*)^* = T$

(e) $\|T^* T\| = \|T T^*\| = \|T\|^2$

f $T^* T = 0 \iff T = 0$

g) $(ST)^* = T^* S^*$

$0^* = 0 \quad I^* = I$

$(T^*)^{-1} = (T^{-1})^*$

Self adjoint, unitary and normal operator

A bounded linear operator $T: H \to H$ on a Hilbert space H is said to be

Self adjoint Hermitian $T^* = T$

unitary is bijective. $T^* = T^{-1}$

normal $\Rightarrow TT^* = T^*T$

V V V. Imp

every finite topological space compact

every countably compact metric space compact