(Norm ... linear ... nail hota) v.v.v. Imp

Hahn Banach Normed Space

→ is an extension for linear Functional

def **Extension of a functional**

let f be a functional defined on a subset Z of a vector space X. A function $\tilde{f}: X \to R$ is to extension of f from Z to X if

$$\tilde{f}(x) = f(x) \quad x \in Z$$

e.g

let $A = \{1, 2\} \subset R$ and $f: A \to R$

$$f(1) = 1 \quad f(2) = 4$$

then $\tilde{f}: R \to R$ defined $\tilde{f}(x) = x^2$ is an extension of f from A to X

$$\tilde{f}(1) = 1 = f(1) \quad \tilde{f}(2) = 4 = f(2)$$

Sub linear function

is a real-valued functional $P: X \to R$ where X is any vector space which satisfies the condition

(i) Sub additive

$$P(x+y) \le P(x) + P(y) \quad x, y \in X$$

(ii) positive homogeneous

$$P(\alpha x) = \alpha P(x) \quad \alpha \geq 0 \text{ in } R, \; x \in X$$

e.g

v.v.v Norm function $\|\cdot\|: X \to R$ is a Sub linear Functional $x, y \in X$ and $\alpha \geq 0$ in R

① $||x+y|| \leq ||x|| + ||y||$

$||\alpha x|| = |\alpha| \, ||x||$

Hahn Banach Therem

Let X be a real valued or complex vector space and p real valued functional on X which is Sub additive $x, y \in X$

① $p(x+y) \leq p(x) + p(y)$

② $p(\alpha x) = |\alpha| p(x)$

Furthermore: let f be a linear functional which is defined on a Subspace Z of X and satisfy

$$|f(x)| \leq p(x) \qquad x \in Z$$

⇒ Hahn Banach says nothing directly about continuity

Hahn Banach statement of theorem

Let f be bounded L-Functional on a Subspace Z of a normed space X. Then there exist a bounded linear functional $\tilde{f}$ on X which is and extension of f to X and has the same norm

$$||f||_Z = ||\tilde{f}||_Z$$

The
⇒

let X be a normed Space $x_0 \neq 0$ be any element of X. Then there exist a bounded linear function $\tilde{f}$ on X ∋ $\|\tilde{f}\| = 1$ and $\tilde{f}(x_0) = \|x_0\|$

The
⇒ Norm zero vector

For every x in a normed Space X

$$\|x\| = \sup_{\substack{f \in X' \\ f \neq 0}} \frac{|f(x)|}{\|f\|}$$

Th
⇒ If x_0 is ∋ $f(x_0) = 0$ for all $f \in X'$ then $x_0 = 0$

The
⇒ for every x_0 in a normed Space X there is a bounded linear functional ∋ $J(x_0) = \|J\| \|x_0\|$ and $\|J\| = \|x_0\|$

Reflexive normed Space ? 26/2|

Isomorphise

$T : X \to Y$

① linear $T(\alpha u + \beta v) = \alpha(T(u)) + \beta(T(v))$

② Bijective

③ norm preserving $\|T(x)\| = \|x\|$

then X is IsomoR