Theorem

Banach fixed point theorem

let (X, d) be complete metric space and $T: X \to X$ is a contraction on X Then T has a unique fixed point

1. Iterative seq $x_{n+1} = Tx_n \quad n = 0,1,2$
2. $d(x_{n+1}, x) \leq \alpha^n (x_n, x_{n-1})$
3. Cauchy seq
4. Fixed point of T
5. uniqueness of fixed pt

Lema

Iteration error bounded

$x_0 \in X$ converges to the unique fixed point of T

$$d(x, x_n) \leq \frac{a}{1-\alpha} d(x_0, x_1)$$

V.V.G) Kannan Fixed point theorem

The let (X, d) be a complete metric space and $T: X \to X$: If there exist some real no α with

$0 < \alpha < \frac{1}{2}$ $\exists$

$$d(Tx, Ty) \leq \alpha(d(x, Tx) + d(y, Ty))$$

T has unique fixed point

The let (X, d) be complete m. Space and $T: X \to X$ If there exist

Hilbert space is always banach space but converse not hold

real no K with $0 < K < 1$

$$d(Tx, Ty) \leq K \max(d(x, Tx), d(y, Ty))$$

V.V.V.V. Imp result

=) A normal space is locally compact iff it is finite dim

=) A normed space is ~~locally~~ finite dim iff its dual space is finite dim

=) every infinite dim normed space has a subspace which is not closed

=) every countable compact metric space is pseudo compact / compact

= The relation of orthogonality in Hilbert space is symmetry

=) A norm defined on a linear space

=) every normed space is banach space iff its is complete

every inner space

=) every Totally bounded metric space is separable

=) x and y are closed subspaces of hilbert space H $\ni x \perp y$ then $x + y$ is closed subspace in H