$[L \neq 15]$

(4) l_∞ = Set of all bounded Seq

$\|x\|_\infty = \sup_{i=1}^{\infty} |x_i|$

$C[a,b] : [x : [a,b] \xrightarrow{\text{conti}} R]$

$x \in C[a,b]$

$x = x(t) \quad t \in [a,b]$

$\|x\| = \max |x(t)| \quad t \in [a,b]$

Results ✓ ✓ ✓ ✓ Imp

The ⟹ ① every normed space is also a metric space but converse is not true e.g $(X, \|\cdot\|)$

(i) $d(x+z, y+z) = d(x,y)$

(ii) $d(\alpha x, \alpha y) = |\alpha| d(x,y)$

This condition for check the norm space

counter example

$X \neq \varphi$

$$d(x,y) = \begin{cases} 1 & x \neq y \\ 0 & x = y \end{cases}$$ is metric space but is not induced by any norm

⟹ discrete metric is not a normed space

The ⟹ Norm $\|\cdot\| \; X \to R$ is a continuous function

$x_n \in X$ with $x_n \to x \in X$

$f(x_n) \to f(x)$ then continuous

⟹ $\|x+y\| \leq \|x\| + \|y\|$

⟹ $\|x\| = \|x - y + y\| \leq \|x - y\| + \|y\|$

$\|x\| - \|y\| \leq \|x - y\|$

$|\|x\| - \|y\|| \leq \|x - y\|$