Bounded linear Functional

Norm

The norm $\|\cdot\| \Rightarrow f: X \to R$ on a normed space $(X, \|\cdot\|)$ is a functional on X but it is not linear because the equality $\|x+y\| \leq \|x\| + \|y\|$ may not hold

Dot product

Let $\alpha = (\alpha_1, \alpha_2, \alpha_3) \in R^3$ be fixed vector Define

$R^3 \to R$ by $= x \cdot \alpha = x_1\alpha_1 + x_2\alpha_2 + x_3\alpha_3$

Then f is bounded linear functional and $\|f\| = |\alpha|$

Integral

① Let $X = C[a,b]$ and $f: X \to R$

$$f(x) = \int_a^b x(t)\,dt \quad x \in C[a,b]$$

then f is bounded linear functional and $\|f\| = b-a$

② Let $X = C[a,b]$ " "

" " " $t_0 \in J = [a,b]$

$f(x) = x(t_0) \quad x \in [a,b]$

f is bounded linear Functional and has norm $\|f\| = 1$.

Space l_2

This series is absolute converges and f is bounded

⇒ Field of real no R and C both also are normed Space

⇒ b.L.F are special classes of Bounded linear operator

The
⇒ If a normed space X finite dim then every linear functional on X is bounded

The
⇒

continuity and boundedness

let $f / D(f) \to Y$ be a L.O where D(f) is Subspace of X and Y, X are normed space then.

(a) f is continuous iff f is bounded

(b) If f is continuous at single point it is continuous

V.V.
⇒ Every dual space is complete

⇒ All norm on X define the same topology.

⇒ dual spaces of L_1 space is L_∞

⇒ If in a normed space X, M = ~~compact~~ $\{x : \|x\| = 1\}$ is compact then X is finite dim space.