

Proof

Suppose that M is continuous

let

$S \subset Y$ be open and

So the inverse of S

of

$S_0 = \varnothing$, it is open

let

$S_0 \neq \varnothing$

For any

$x_0 \in S_0$ let $y_0 = Mx_0$

Since

S is open

it contains an ϵ -neighborhood N
of y_0

Since M is continuous

x_0 has a δ -neighborhood N_0 which
is mapped into N

Since $N \subset S$ then $N_0 \subset S_0$

So that

S_0 is open because $x_0 \in S_0$

Conversely

assume that the

inverse image of every open set